# A New Stem Volume Equation for Norway Spruce (*Picea abies* (L.) Karst) Trees in Romania

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*Abstract*

A new equation for volume estimation of tree stems was developed using median diameter of stem profile. This work is based on the volume equation for solids of revolution defined originally by Mathiesen in 1925. The study is based on previously published data for 5,403 Norway spruce trees from Romania grouped in 218 height/diameter categories. For each of these categories, median diameter and area of stem profile as well as volumes by Mathiesen's formula and Huber's formula were computed. The volume equation was obtained using quick methods for determining the median diameter and the area of stem profile. The proposed model provides more accurate results than the original equation developed by Mathiesen. Furthermore, it allowed the development of an easy-to-use equation aiming to predict the over-bark stem volume, based on the median diameter of stem profile.

**Keywords:** model, Norway spruce, median diameter, stem area, stem volume

## Introduction

There are several methods used to estimate tree volume either for felled or standing trees. Smalian's, Huber's and Newton's formulae are commonly used to estimate the volume of different tree sections (frustums) after tree felling and measuring the lengths and diameters on the tree bole sections obtained. The formulae are applied assuming that bole sections can be assimilated into frustum of paraboloid and cylinder (Philip 1994, Giurgiu et al. 2004, Leahu 2007, Tomusiak and Zarzynski 2007, van Laar and Akça 2007, Cruz de Leon 2010, Soares et al. 2010). Various formulae based on three predictors (diameter, height and form) are recommended for standing trees in forestry literature (Schiffel 1899, Pollanschütz 1965, Giurgiu 1979, Avery and Burkhart 1983**,** Philip 1994, Husch et al. 2003, Parent and Moore 2003, Giurgiu et al. 2004, Leahu 2007, Tomusiak and Zarzynski 2007, van Laar and Akça 2007). Usually, research employs total tree volume equations that use the diameter at breast height and tree height as independent variables (Perez 2008, VanderSchaaf 2008, Petrauskas and Rupsys 2010, Rupsys and Petrauskas 2010, Burkhart and Tome 2012, Yousefpour et al. 2012).

Among the earliest approaches of volume estimation, it is that based on the use of the form point concept, which represents the centre of wind resistance located at the centre of gravity of the tree crown. The relative height above the ground of this form point was firstly used by Jonson in 1928 (van Laar and Akça 1997). Later it was replaced by more efficient estimators of stem volume.

Some equations make use of sections other than cross-sectional area at the midpoint or cross-sectional area at the lower and upper ends. One of them is described as the centroid/centre of gravity method (Forslund 1982, Wiant et al. 1991, Yavuz 1999, Coble and Wiant 2000, Wiant et al. 2002, Coble and Lee 2003, Ozcelik et al. 2006). The importance of using the cross-sectional area at 1/3$^{rd}$ of the stem for volume estimation was described by Hossfeld (Giurgiu 1979, Husch et al. 2003, van Laar and Akça 2007, Leahu 2007, West 2009, Ducey and Williams 2011, Durkaya and Durkaya 2011). A similar theory involves the use of true form quotient depending on diameters above ground at $0.1\times h$ and $0.3\times h$, where $h$ is tree height (Pollanschütz 1965, Giurgiu et al. 2004).

Mathiesen's formula (Eq.1) for stem volume ($v$) estimation (Mathiesen 1925) is not commonly used worldwide. It was developed in 1925 and it is based on the median diameter ($dm$) and total area ($S$) of stem profile. $S$ is defined as the area of the stem longitudinal section and $dm$ is the diameter, which divides the stem profile in two parts of equal area (Figure 1).

$$v = \frac{3}{4} \times dm \times S \quad (1)$$

In principle, the method supposes that the stem centre of gravity is located at the median diameter (Figure 1).

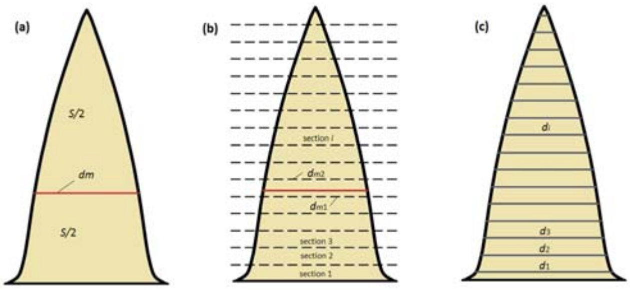


**Figure 1.** Median diameter of the stem area (**a**) position on the stem profile; (**b**) stem divided in sections and diameters of the section ends where median diameter is located; (**c**) diameters along the stem used to compute the area (*dm* – median diameter of stem profile, *S* is the area of stem profile, $d_{m1}$ and $d_{m2}$ are diameters of the section ends, where *dm* is located, $d_i$ are the diameters at the middle of sections)

The area of stem profile is based on Guldin's theory of volumes and surface areas of solids of revolution and on centres of gravity of bodies (Ichim 1954, Leahu 2007). Initially, Mathiesen proposed Eq. 2 as a formula for stem volume, where $l$ is length of stem sections and $d_i$ are diameters at the middle of sections (Mathiesen 1925, Leahu 2007). In this case the area of stem profile is expressed by summing up the longitudinal section area ($l \times d_i$) of each section.

$$\upsilon = \frac{\pi}{4} \times dm \times l \times \sum_{i=1}^{n} d_i \qquad (2)$$

Due to the fact that this formula overestimates stem volume, the value of the ratio $\pi/4$ (0.7854) was reduced to 3/4 (0.75) as in Eq. 1. Despite this change Mathiesen's formula wasn't applied in practice because of its laborious elements. It requires the measurement of stem diameter at the middle of sections followed by the computation of stem area and median diameter. However, the latter does not refer to the median value of diameters along the stem. This approach of volume determination is still unknown in many countries. History of Estonia explains why Mathiesen's researches from university of Tartu (Estonia) are known in Baltic States, Germany and old Soviet Union, especially. For these reasons Huber's formula (Eq. 3) was preferred for a more accurate volume quantification of the tree stem.

$$\upsilon = \frac{\pi}{4} \times l \times \sum_{i=1}^{n} d_i^2 \qquad (3)$$

There are many pro and contra arguments when comparing the Huber's formula with the other two formulae commonly used to estimate the volume of tree sections. For instance, West (2009) argued that the Newton's formula gives more accurate estimates compared to Huber's and Smalian's formulae, because it uses more information to calculate the volume. However, the Smalian's formula is easier to use, due to its practicability. Accuracy of the volume estimates is affected by both, the section length and the section position between the base and the top of the tree. Most of the direct stem diameter measurements are made using short lengths (0.5- 1 m) in the case of large trees (West 2009). Soares et al. (2010) found a mean difference of 1.23% when estimating the tree volume using 1 m sections compared to 2 m sections and of 3.5% when using 1-m sections compared to 3-m sections. Akossou et al. (2013) found that the Huber's formula estimates in an improved way the entire stem volume when using sections spaced at 0.5 m and the stem base volume when using sections of 1, 2 and 3 m. They also suggested the use of Smalian's and Newton's formulae for the remaining part of the stem, excepting the first section from the base. Nevertheless, the Newton's formula is rarely used in practice (Philip 1994).

The idea to develop a volume equation which uses the breast height diameter, median diameter of stem and total height of trees as variables originated during previous research on median diameter along stem profile (Vasilescu et al., in prep.). The work emphasizes that in the case of Norway spruce trees (*Picea abies* (L.) Karst) the median diameter of stem profile (*dm*) can be determined by quick methods. Firstly, in case of felled trees, usually it represents the value of diameter at $0.3 \times h$ distance away from tree base ($d_{0.3h}$). Secondly, in case of standing trees, median diameter of stem profile can be computed with a multiple regression having the breast height diameter and the total tree height as independent variables.

These results enable the possibility to use in practice Mathiesen's formula for stem volume estimation of standing and felled trees without extensive measurements of diameters along the stem. However, such an approach would require accurate models to estimate the area of stem profile. Using actual possibilities for computing and a large database elaborated for Norway spruce trees, we proposed the following objectives:

1) to analyze the values of stem volume obtained by applying Mathiesen's formula;

2) to improve Mathiesen's formula for a quick field applicability and for better results for stem volume.

## Methods

A database was generated from 5,403 stem analysis data of Norway spruce trees from 200 different sites in Romania (Popescu-Zeletin et al. 1957, Vasilescu 2013). Trees breast height diameter varied between 12 and 60 cm and total height between 10 and 42 m. These trees spanned across 218 diameter-height categories and average values of the stem diameter at every 2 meters from the breast height diameter were computed for each category. The database used consisted of 3,044 pairs of over-bark stem diameter - height values. Based on these values the present work followed the next four stages of research:

I. computing the area of stem profile ($S$) and median diameter ($dm$) of stem for each tree of the 218 categories;

II.determining stem volume of Norway spruce trees using Huber's and Mathiesen's formulae;

III. determining the ratio between the volume computed with Huber's formula and the volume obtained by multiplying the median diameter with area of stem profile;

IV. elaborating a new equation, computing stem volume with the new equation and analysis of volume accuracy.

The area of stem profiles was computed using Eq. 4, where $l$ is the length of stem sections ($l$ = 2 m) and $d_i$ is the diameter at the middle of sections $i$ ($i$ =1, 2, ..., $n$). Eq. 4 is based on the summation of the longitudinal sectional area ($l \times d_i$) of each section (Figure 1). As the uppermost part of the stem contributes very little to the total area and the total height of trees was assigned to 2-meter in size classes, a lesser accuracy of the top estimation was considered to be insignificant when computing the area. It was also important to use the same diameters in both, the Mathiesen's and Huber's formulae to keep the computation consistency (e.g. avoiding other sources of differences). Shorter lengths (e.g. 1 m) could be necessary for the small trees (less than 10 meters in height). In such cases the area of the top should be computed as the area of a triangle and added to the total area.

$$S = l \times \sum_{i=1}^{n} d_i \quad (4)$$

These results helped to determine the values of median diameter of stem profile computed with Eq. 5, where $S_{m1}$ and $S_{m2}$ are areas with values above and below $S$/2 in the series of cumulated areas, and $d_{m1}$ and $d_{m2}$ (Figure 1) are corresponding values of diameters of $S_{m1}$ and $S_{m2}$, respectively.

$$dm = d_{m1} - (d_{m1} - d_{m2}) \times \frac{\frac{S}{2} - S_{m1}}{S_{m2} - S_{m1}} \quad (5)$$

The best fitted model that could be used to estimate median diameter using two independent variables, breast height diameter ($dbh$) and tree height ($h$) is given in Eq. 6.

$$dm = a_0 + a_1 \times dbh + a_2 \times dbh^2 + a_3 \times h + a_4 \times h^2, \quad (6)$$

where the coefficients for Norway spruce trees are $a_0$ = 1.2792, $a_1$ = 0.7533, $a_2$ = -0.0012, $a_3$ = -0.0050 and $a_4$ = 0.0014.

A confidence level of 95% was used to estimate the coefficients of the model enclosed in Eq. 6 while the coefficient of determination was of 0.9996. Also, the standard error of the estimate was of 1.8 mm.

Median diameter multiplied with area of stem profile gives the volume expression of a body, an idea that is generally accepted. Despite reducing the value of $\pi$ (from Eq. 2) to 3.0 (as in Eq. 1), Mathiesen's formula was still criticized for generally overestimating the stem volumes. To be able to improve this formula, we calculated the ratio ($pv$) between the volume computed with Huber's formula and the volume obtained by multiplying the median diameter with area of stem profile (Eq. 7).

$$pv = \frac{V_{Huber}}{dm \times S} \quad (7)$$

In this study, the use of Huber's formula seemed to be a good choice because it yields small errors, when there is measurement error for $d_i$ and $l$ (Biging 1988). Compared to the Newton's and Smalian's formulae, it returns more accurate results for volumes of those sections located at the stem base (West 2009, Akossou et al. 2013). This would be very important since in the case of the studied trees the first section from the stem base includes a volume varying between 11% and 38% of the total stem volume. By using the sections of 2 meters to estimate the total stem volume we assumed an error of 1% as earlier mentioned by Altherr (Akossou et al. 2013). Moreover, such errors tend to be zero for the central part of the stem and very low for the last two sections from the tip.

Scientifically, the Huber's formula was used in Romania to estimate the volume of round wood (Popescu-Zeletin et al. 1957, Giurgiu et al. 1972, Giurgiu et al. 2004). Also, its use in practice is required by law. The first Romanian tree volume estimation tables were developed by Popescu-Zeletin et al. (1957) based on a large database (38,533 trees) for a number of 17 species using the Huber's formula. Later the database was expanded and Giurgiu et al. (1972) developed additional tree volume estimation tables for another 11 species. In their revised edition the volume estimates of Norway spruce trees were taken from Popescu-Zeletin et al. (1957).

In order to develop a more accurate model to be used for stem volume estimation, we chose from an extensive set of models the best fitted ones predicting $S$ and $pv$ as functions of two (breast height diameter and tree height) or three (breast height diameter, median diameter of stem area and tree height) independent variables. The final model represents a variation of the Mathiesen's original formula that is able to estimate stem volume of Norway spruce trees more accurately and in a more effective manner.

Microsoft Office Excel and Statistica 9 software were used for data analysis. The method of least squares was utilized to compute parameter estimates. The coefficient of determination ($R^2$) was computed to measure how well the model fits to the data. Afterwards the analysis of residuals was carried out in order to estimate the accuracy of predicted values. Also, the sum of squared errors (*SS*) and the root mean squared error (*RMSE*) were computed to select the best fitted model for stem volume of fell and standing trees. The model validation was implemented using a data subset systematically extracted from the original data pool.

As the median diameter of stem area described better the shape of tree stem by comparison with the breast height diameter, it was used as a third variable to express the area and volume of stem. This assumption is demonstrated by Figure 2, where the plots of relative height and relative stem diameter over-bark are shown for all of the 3,044 pairs. Firstly, the relative values were computed using tree height and breast height diameter. Secondly, the relative stem diameter was obtained by dividing stem diameter by median diameter of stem area. The dispersion was greater when breast height diameter was used to describe stem shape.

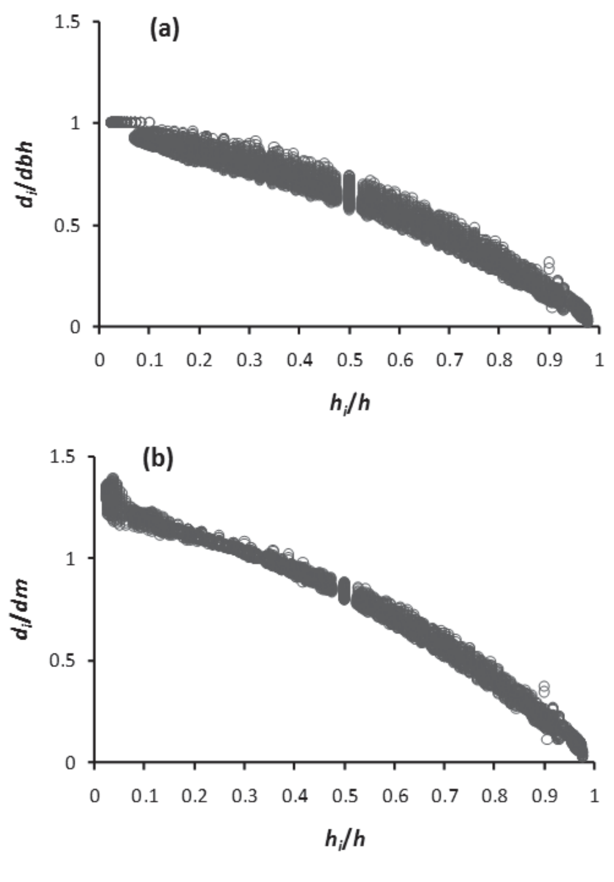


**Figure 2.** Plot of relative height for Norway spruce trees in Romania versus relative over-bark diameter computed with (**a**) breast height diameter; (**b**) median diameter of stem area (*dbh* is the breast height diameter, *dm* is the median diameter of stem profile, $d_i$ are diameters along stem at the middle of sections, *h* is the tree height, $h_i$ are heights at the middle of sections)

Modern software, like many of those currently used in forest and range management, allow for a quick graphic determination of size of an area and the position and size of the median diameter of a shape profile (Tereşneu and Ionescu 2011, Tereşneu 2012).

## Results

Figure 3 shows the relationship between tree characteristics and area of stem profile. The graph emphasizes the variation of longitudinal area of stem with breast height diameter, median diameter of stem profile and tree height.

According to Figure 3, the best model is represented by a power function depending on median diameter. However, the coefficient of determination between these variables is also very high (i.e. 0.903). If we use a multiple regression (Eqs. 8-11) it yields a easier and more precise relationship to use in the field.

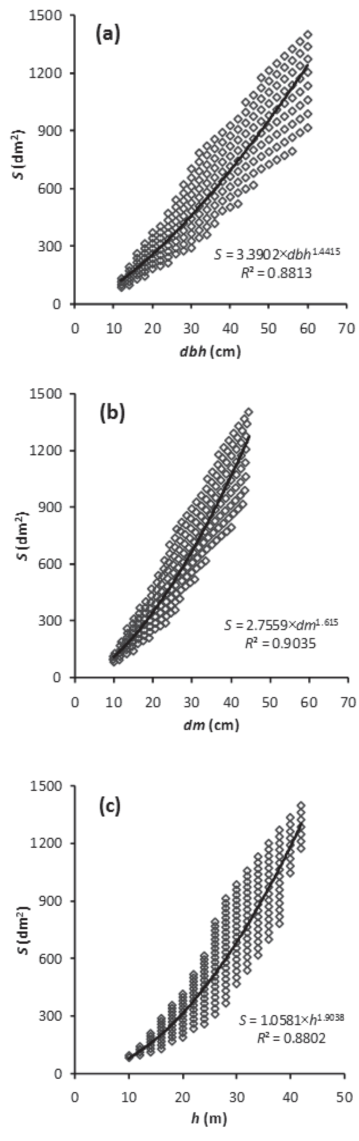


**Figure 3.** Stem area for Norway spruce trees in Romania varying as a function of (**a**) breast height diameter; (**b**) median diameter of stem area; (**c**) tree height (*S* is the area of stem profile, *dbh* is the breast height diameter, *dm* is the median diameter of stem profile, *h* is the tree height)

$$S = b_0 + b_1 \times dbh + b_2 \times dbh^2 + b_3 \times h + b_4 \times h^2 + b_5 \times dm + b_6 \times dm^2 \quad (8)$$

$$S = b_0 + b_1 \times dbh + b_2 \times dbh^2 + b_3 \times h + b_4 \times h^2 \quad (9)$$

$$\log S = b_0 + b_1 \times \log dbh + b_2 \times \log^2 dbh + b_3 \times \log h + b_4 \times \log^2 h + b_5 \times \log dm + b_6 \times \log^2 dm \quad (10)$$

$$\log S = b_0 + b_1 \times \log dbh + b_2 \times \log^2 dbh + b_3 \times \log h + b_4 \times \log^2 h \quad (11)$$

The estimated coefficients and fit statistics for the equations 8-11 are given in Table 1.

**Table 1.** Parameter estimates (coefficients) and fit statistics of the stem area estimated models

| Statistics of the estimated models | Symbols | Estimated equations of stem area (dm$^2$) | | | |
|---|---|---|---|---|---|
| | | Three variables, *dbh* (cm), *h* (m) and *dm* (cm) | | Two variables, *dbh* (cm) and *h* (m) | |
| | | *S* (Eq. 8) | log *S* (Eq. 10) | *S* (Eq. 9) | log *S* (Eq. 11) |
| Coefficients ± standard errors | $b_0$ | 92.5073 ±12.99483 | -0.014832 ±0.001694 | 5.26648 ±0.812161 | -0.008853 ±0.001776 |
| | $b_1$ | 45.0517 ±5.08049 | 0.667928 ±0.297442 | 6.50545 ±0.58355 | 1.135432 ±0.0207 |
| | $b_2$ | -0.7007 ±0.0586 | -0.066232 ±0.010663 | 0.09418 ±0.00732 | -0.105263 ±0.006888 |
| | $b_3$ | 1.4335 ±0.296771 | 0.74292 ±0.043506 | -7.2897 ±1.07605 | 0.67486 ±0.028783 |
| | $b_4$ | 0.3461 ±0.0189 | 0.11049 ±0.016862 | 0.55221 ±0.01856 | 0.144376 ±0.010354 |
| | $b_5$ | -66.8896 ±7.61593 | 0.441799 ±0.134309 | | |
| | $b_6$ | 1.7316 ±0.12266 | -0.013115 ±0.005133 | | |
| Coefficient of determination | $R^2$ | 0.9989 | 0.9999 | 0.9977 | 0.9998 |
| Sum of squared errors | *SS* | 27,215.24 | 0.001666 | 56,416.42 | 0.001985 |
| Root mean squared error | *RMSE* | 11.35703 | 0.002809 | 16.2747 | 0.003053 |

By using any of these expressions in the Mathiesen's formula, we can substitute the area of stem profile ($S$), which is more difficult to compute. Combined with the use of a quick method for determining the median diameter (i.e. a method not using the profile area or many stem diameters measurements) this method provides a more efficient way for volume estimation.

The percentage residuals of stem area estimated with Eqs. 8-11 are shown in Figure 4. As shown, the best results for predicting the stem area may be obtained using a logarithmic fitting (Eqs. 10 and 11). In order to use of Eq. 10 data on breast height diameter, tree height and median diameter needs to be acquired by measurement. In the case of felled trees, the median diameter can be measured at $0.3 \times h$ relative to the tree base. In the case of standing trees, the median diameter can be measured only if adequate measuring equipment is available or it can be estimated using Eq. 6. Similar results, even if less accurate, can be obtained when the stem area is estimated using Eq. 11. The use of the latter always supposes the measurement of both, the breast height diameter and the tree height.

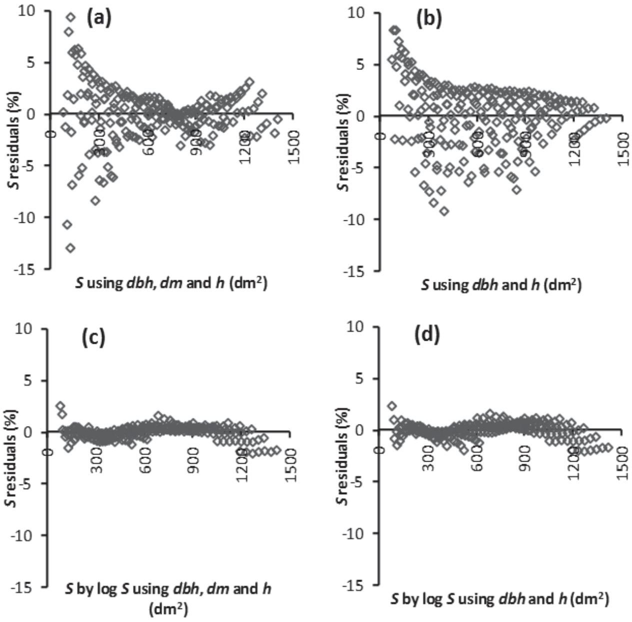


**Figure 4.** Percentage residuals of stem area estimated with (**a**) three variables using Eq. 8; (**b**) two variables using Eq. 9; (**c**) three variables using a logarithmic function (Eq. 10); (**d**) two variables using a logarithmic function (Eq. 11); (*S* residuals are percentage residuals of stem area, *dbh* is the breast height diameter, *dm* is the median diameter of stem profile, *h* is the tree height)

Then, the volume proportion ($pv$) that was computed using Eq. 7 has been analyzed in order to establish the measure of coefficient from the Mathiesen's equation. The obtained results are graphically shown (Figure 5) as a function of breast height diameter, median diameter and the tree height. As shown, the variation of $pv$ values is rather low and they cannot be accurately estimated using a single variable. The values of volume proportion are less dispersed for very tall trees compared to small ones.

Two equations can be recommended for the estimation of volume proportion as functions of the dimensional characteristics of the tree: Eq. 12 with three predictors (*dbh*, *h* and *dm*) and Eq. 13 with two predictors (*dbh* and *h*).

$$pv = c_0 + c_1 \times dbh + c_2 \times dbh^2 + c_3 \times h + c_4 \times h^2 + c_5 \times dm + c_6 \times dm^2 \quad (12)$$

$$pv = c_0 + c_1 \times dbh + c_2 \times dbh^2 + c_3 \times h + c_4 \times h^2 \quad (13)$$

The parameter estimates and fit statistics for the equations are given in Table 2.

The percentage residuals of the volume ratio as generated by Eq. 12 are shown in Figure 5 (d). In order to get a unique value of the volume proportion, the volume as it was calculated using the Huber's formula was estimated as a function of the median diameter and stem area using Eq. 14.

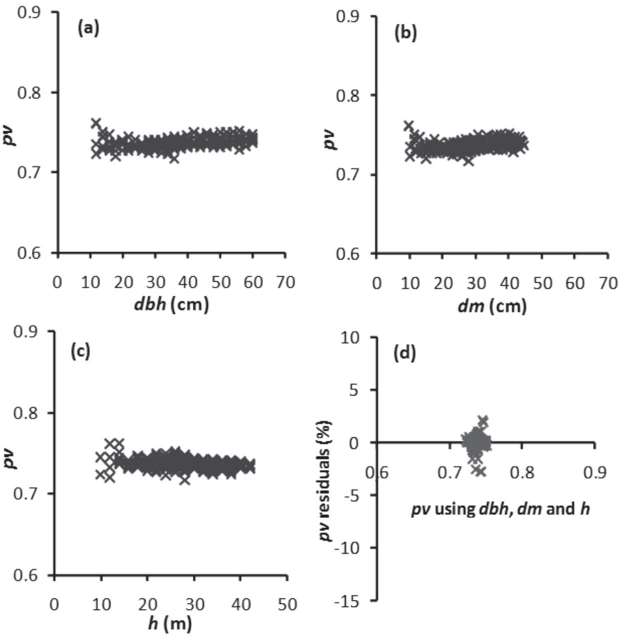


**Figure 5.** Proportion of volume related with (**a**) breast height diameter; (**b**) median diameter of stem area; (**c**) tree height and percentage residuals of volume ratio (**d**) estimated with three variables (Eq. 12) (*pv* is the proportion of volume, *dbh* is the breast height diameter, *dm* is the median diameter of stem profile, *h* is the tree height)

**Table 2.** Parameter estimates and fit statistics of the regression equations

| Statistics of the estimated models | Symbols | Parameter estimate for the regression models: Three variables, *dbh* (cm), *h* (m) and *dm* (cm) (Eq. 12) | Two variables, *dbh* (cm) and *h* (m) (Eq. 13) |
|---|---|---|---|
| Coefficients ± standard errors | $c_0$ | 0.771234±0.004051 | 0.744029±0.003372 |
| | $c_1$ | 0.015318±0.001584 | 0.000209±0.000162 |
| | $c_2$ | -0.000073±0.000018 | 0.000003±0.000000 |
| | $c_3$ | -0.000104±0.00003 | -0.000644±0.000298 |
| | $c_4$ | 0.000007±0.000000 | -0.000002±0.000000 |
| | $c_5$ | -0.021174±0.002374 | |
| | $c_6$ | 0.000123±0.000038 | |
| Coefficient of determination | $R^2$ | 0.7013 | 0.5104 |
| Sum of squared errors | *SS* | 0.002645 | 0.004336 |
| Root mean squared error | *RMSE* | 0.003540 | 0.004512 |

$$v = \alpha_0 \times dm \times S \quad (14)$$

The coefficient of this equation resulted by considering the stem volume in dm$^3$, median diameter in dm and the stem area in dm$^2$ and it is $\alpha_0 = 0.73819$. The coefficient of determination is equal to 0.9998, the sum of squared errors is equal to 37,301.14 and the root mean squared error is equal to 13.11. This analysis allows the adjustment of Mathiesen's formula (Eq. 15).

$$v = 0.7382 \times dm \times S \quad (15)$$

When applying the new equation to all of the 218 categories, the results showed a similar dispersion to that obtained by using the Mathiesen's formula.

Differences between volumes using Mathiesen's formula and Huber's formula are up to 5%. Figure 6 (a) shows that the Mathiesen's formula most often leads to the volume over-estimation producing biased results.

By reducing the value of the scaling factor of Mathiesen's formula from 0.75 to 0.738, the results are closer to those obtained with Huber's formula. This is obvious in Figure 6 (b), where volume residuals are grouped on *Ox* axis. In addition, the practical use of Eq. 15 generates unbiased results. However, the effect is good for estimating the volume for big trees.

The percentage residuals of volume are more efficient (Giurgiu 1972) to quantify the effect of using Mathiesen's formula and the modified expression also (Figure 6). The volume by Huber's formula was considered reference value (Biging 1988) in all situations.

Mahiesen's formula can be generalized using Eq. 16.

$$v = pv \times dm \times S \quad (16)$$

By applying Eq. 16 for Norway spruce trees in Romania, the percentage residuals of stem volume vary as a function of the used estimation equations. In Figure 6 (c), it is shown the effect of using Eqs. 6, 8 and 12 to estimate variables from Eq. 16. If Eqs. 6, 9 and 13 are used to estimate the variables from Eq. 16, then the percentage residuals of volume have a greater dispersion as shown in Figure 6 (d).

The use of logarithmic equation to express the stem area leads to improved results in both tested variants as shown in Figure 6 (e and f). Figure 6 (e) shows the effect of using Eqs. 10 and 12, when the median diameter can be measured on the tree stem. In Figure 6 (f) it is emphasized the cumulated effect of replacing the variables from Eq. 16 with estimation produced using Eqs. 6, 11 and 13 as well as by the use of measurements of breast height diameter and tree height.

Therefore, the classical calculations of median diameter and stem area using the Mathiesen's formula can be replaced with good results by estimations as functions of tree dimensional characteristics (*dbh* and *h*). The stem volume that is to be estimated using the aforementioned procedures yields more accurate results in comparison with the use of volume tables. In Romania, the volume tables are available for 43 tree species (Giurgiu et al. 2004) and are based on the logarithmic function given in Eq. 17.

$$\log v = \alpha_0 + \alpha_1 \times \log dbh + \alpha_2 \times \log^2 dbh + \alpha_3 \times \log h + \alpha_4 \times \log^2 h \quad (17)$$

For the Romanian Norway spruce the coefficients of volume estimates (m$^3$) using Eq. 17 and the existing database have been produced by Giurgiu et al. (2004) and they have the following values: $\alpha_0$ = -4.18161, $\alpha_1$ = 2.08131, $\alpha_2$ = -0.11819, $\alpha_3$ = 0.70119 and $\alpha_4$ = 0.148181. The percentage residuals produced by the use of this equation are shown in Figure 6 (g). It can be observed that the practical use of this equation may lead to biased results in the case of very small trees having a volume less than 0.5 $m^3$ (under-estimation of up to 3%) as well as for those having a volume greater than 2 $m^3$ (over-estimation of up to 3%). For those trees having a volume between 0.5 and 2 $m^3$, the use of model enclosed in Eq. 17 leads to unbiased results. Therefore, it can be specified that the models enclosed in Figure 6 (e and f) are able to produce improved effects (unbiased results) if compared to the

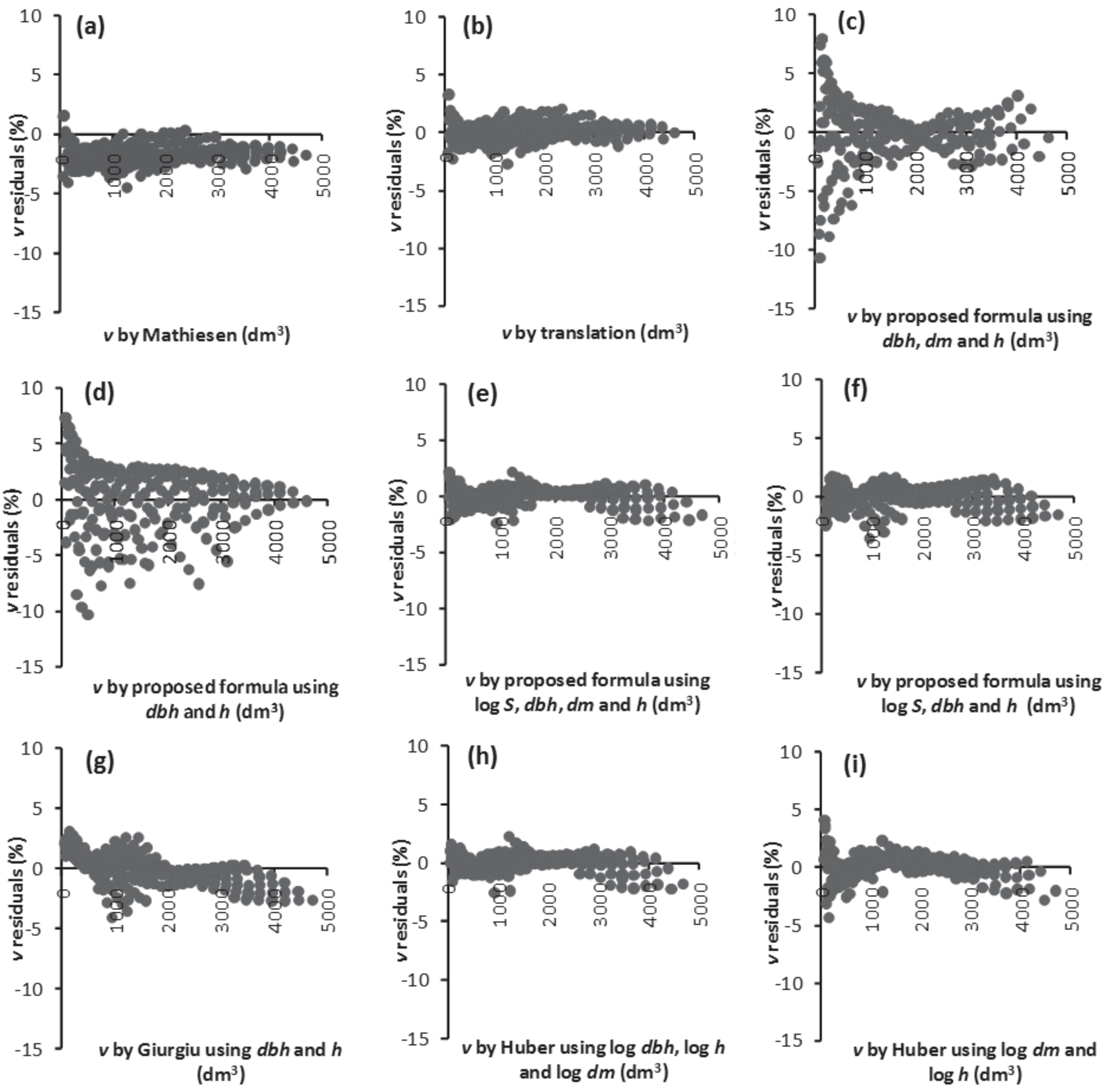


**Figure 6.** Percentage residuals of stem volume for Norway spruce trees in Romania using (**a**) Mathiesen's formula and classical computation (Eq. 1); (**b**) appropriate coefficient of Mathiesen's formula (Eq. 15); (**c**) proposed formula with three variables (Eqs. 6, 8 and 12); (**d**) proposed formula with two variables (Eqs. 6, 9 and 13); **e**) proposed formula and a logarithmic function with three variables (Eqs. 10 and 12); (**f**) proposed formula and a logarithmic function with two variables (Eqs. 6, 11 and 13); (**g**) Giurgiu's equation with two variables (Eq. 17); (**h**) volumes estimated with three variables and a logarithmic function (Eq. 18); (**i**) volumes estimated with two variables and a logarithmic function (Eq. 19); (*v* residuals are percentage residuals of stem volume, *dbh* is the breast height diameter, *dm* is the median diameter of stem profile, *h* is the tree height)

currently used model in the Romanian volume tables. The increased accuracy of the models enclosed in Figure 6 (e and f) may be explained by the greater number of coefficients and the proper choice of equations. Given the ability of logarithmic function to reproduce more accurately the estimation model of stem area and volume, there were proposed Eqs. 18 and 19.

$$\log v = \alpha_0 + \alpha_1 \times \log dbh + \alpha_2 \times \log^2 dbh + \alpha_3 \times \log h + \alpha_4 \times \log^2 h + \alpha_5 \times \log dm + \alpha_6 \times \log^2 dm \quad (18)$$

$$\log v = \alpha_0 + \alpha_1 \times \log dm + \alpha_2 \times \log^2 dm + \alpha_3 \times \log h + \alpha_4 \times \log^2 h \quad (19)$$

Using these equations, it was possible to emphasize the role of median diameter of stem area in the estimation model of stem volume of Norway spruce trees. The median diameter was added as a supplementary variable (Eq. 18) compared to the equation used to derive volume tables and in Eq. 19 the median diameter has replaced the breast height diameter. Parameter estimates for these equations are given in Table 3.

**Table 3.** Parameter estimates and fit statistics of the stem volume estimated models

| Statistics of the estimated models | Symbols | Estimate equations of stem volume ($dm^3$) | |
|---|---|---|---|
| | | Three variables, *dbh* (cm), *h* (m) and *dm* (cm) (Eq. 18) | Two variables, *dm* (cm) and *h* (m) (Eq. 19) |
| Coefficients ± standard errors | $\alpha_0$ | -1.13078±0.020376 | -1.13714±0.027792 |
| | $\alpha_1$ | 1.79407±0.357705 | 1.93209±0.036963 |
| | $\alpha_2$ | -0.29094±0.128238 | 0.01357±0.001328 |
| | $\alpha_3$ | 0.81328±0.052323 | 0.95721±0.046506 |
| | $\alpha_4$ | 0.08757±0.02028 | 0.01181±0.001674 |
| | $\alpha_5$ | 0.14159±0.041261 | |
| | $\alpha_6$ | 0.27444±0.016084 | |
| Coefficient of determination | $R^2$ | 0.9999 | 0.9998 |
| Sum of squared errors | *SS* | 0.002409 | 0.004904 |
| Root mean squared error | *RMSE* | 0.003379 | 0.004798 |

Eq. 18 works with three variables, viz.: breast height diameter, tree height and the upper stem diameter (median diameter), while Eq. 19 allows the estimation of stem volume as a function of an upper diameter (median diameter) and tree height. The percentage residuals of the stem volume that were obtained using Eqs. 18 and 19 are shown in Figure 6 (h and i). The use of these two models leads to unbiased results as well as to very small differences.

Therefore, the proposed expressions for stem volume (Eqs. 18 and 19) provides a better model for estimating stem volume of Norway spruce trees in Romania.

## Discussion and Conclusions

The use of those equations that require an additional upper stem diameter in order to estimate the stem volume for standing trees is not specific to Romania. However, the results show that by adding the median diameter as a supplementary variable, significant improvements may be achieved in the estimated volumes of individual trees. The median diameter may be associated with $d_{0.3h}$ while in measurements different hand-held tools may be used. In this respect, McCaffery et al. (2015) suggested the use of TruPulse equipment by both experienced and inexperienced users. The same authors recommend Impulse as the best equipment to be used by experienced users, arguing their recommendation based on the measurement time consumption and the equipment cost. Shimizu et al. (2014) proposed an ultra-telephotographic system to be used in the measurement of the upper stem diameter based and developed a new method for such measurements. However, such a system still needs further tests in flatland and sloped forests in order to improve the results, but it has a lot of potential in implementations given its low costs.

The study indicates that Mathiesen's formula can be improved at least for Norway spruce trees in Romania. The new expression (Eq. 15) allows the assessment of stem volume with more accuracy.

In addition, the new formula proposed for the area of stem profile (Eq.10) allows one to transform Eq. 16 to an expression with three (*dbh*, *h* and *dm*) variables (Eq. 20).

$$v = (c_0 + c_1 \times dbh + c_2 \times dbh^2 + c_3 \times h + c_4 \times h^2 + c_5 \times dm + c_6 \times dm^2) \times dm \times 10^{b_0 + b_1 \times \log dbh + b_2 \times \log^2 dbh + b_3 \times \log h + b_4 \times \log^2 h + b_5 \times \log dm + b_6 \times \log^2 dm} \quad (20)$$

The new model which proposes the changes in Mathiesen's formula is improving the volume estimation. Eq. 20 makes it possible to estimate quickly the stem volume ($dm^3$) by measuring breast height diameter, tree height and median diameter of stem profile at $0.3 \times h$.

Applying this equation volume, differences are not significant. In addition, in order to determine the volume of a stem using proposed equation the results are better than volume tables by species. The last produce considerable errors (±16%) (Giurgiu et al. 2004).

In case of standing trees Eq. 6 elaborated by the same authors estimates the median diameter. This is the solution when the diameters at different heights along a tree cannot be measured.

The development of better model for stem volume of Norway spruce was facilitated by the recent results regarding quick methods to assess median diameter for this species. Such a model is very useful taking into consideration the large natural range of the species in Europe. In most European countries the volume equations of Norway spruce trees are given as functions of two variables, *dbh* and *h* (Zianis et al. 2005).

The results of this study support the research of Pollanschütz (1965), who used three variables (*dbh*, *h* and $d_{0.3h}$) to estimate either the form factor or the stem volume. The model developed by Pollanschütz (1965) to estimate the stem volume of Norway spruce trees was adapted by substituting $d_{0.3h}$ with $dm$ (Eq. 21).

$$v = \alpha_0 \times dbh^2 \times h + \alpha_1 \times dbh \times dm \times h + \alpha_2 \times h^2, \qquad (21)$$

where the coefficients (± standard errors) are the following: $\alpha_0$ = -0.012115 (±0.00259), $\alpha_1$ = 0.056199 (±0.003659) and $\alpha_2$ = 0.116926 (±0.011427); $R^2$ = 0.9997, $SS$ = 82,639.75 and $RMSE$ = 19.69721. The adapted model (Eq. 21) produced the best results for trees having a volume greater than 0.5 m$^3$.

This study tested other equations in order to get better volume estimates compared to the last four equations (Eqs. 18-21). The median diameter improved the volume estimates. This was also true when the $dm/dbh$ ratio was used. The model (Eq. 22) was adapted from the three predictor variables ($d_{0.1h}$, $d_{0.5h}$ and $h$) equation developed by Giurgiu et al. (2004).

$$v = dbh^2 \times h \times \left[\alpha_0 + \alpha_1 \times \frac{dm}{dbh} + \alpha_2 \times \left(\frac{dm}{dbh}\right)^2\right], \qquad (22)$$

where the coefficients (± standard errors) are the following: $\alpha_0$ = 0.147673 (±0.025222), $\alpha_1$ = -0.394517 (±0.066356) and $\alpha_2$ = 0.319674 (±0.043625); $R^2$ = 0.9996, $SS$ = 98,328.93 and $RMSE$ = 21.38. Compared to Eqs. 18 and 19 the equation yields unbiased results as well as very small differences. In addition, Eq. 22 has the advantage of using only three coefficients.

Doing to its large number of coefficients Eq. 20 is not easy to use in practice. By considering the accuracy and practicability of the volume equations using the median diameter of the stem as additional variable against the ordinary model having two variables, Eqs. 22 and 21 are recommended for volume estimation. While the Eq. 18 has many coefficients it also yielded the best accuracy.

The new model can be compared also with a taper equation (Biging 1988, Özçelik and Göçeri 2015) elaborated for Norway spruce in Romania.

Replicating this kind of research for other species will help to produce a general and easy to use volume equation for conifers species.

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